Homework Week 3: Gaussian Distribution

**Exercise 1:**

1. Prove that the Univariate Gaussian PDF is normalized.

We have the pdf of GD as:

To prove its normalization is to prove :

First prove it with a mean equal zero function for simplicity :

We have to prove

⬄

Denote (2) = I

Transform into polar coordinates <DOUBLE INTEGRALS IN POLAR COORDINATES>

With

* PDF with zero-mean is proved to be normalized

Transform

* Thus, Univariate Gaussian PDF is normalized.

1. A random variable 𝑋 follows Gaussian distribution (notation: 𝑋 ∼ N(𝜇, 𝜎2)). Prove  
   that the expected value of X is 𝜇 and the standard deviation of X is 𝜎

An implication of X being normally distributed with 𝜇 and is that Z = (X – 𝜇)/ 𝜎 is normally distributed with parameters 0 and 1.

Var [Z] = E[

Let u = x 🡪 du = dx and dv =

Var[Z] =

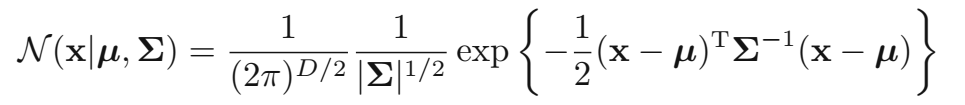
Since X = 𝜎Z + 𝜇 🡪 E[X] = E[𝜇] + 𝜎E[Z] = 𝜇

And Var[X] = 𝜎2Var[Z] = 𝜎2

By definition, the standard deviation equals square root of Var[X] =

**Exercise 2:**

1. Prove that the Multivariate Gaussian PDF is normalized



Define y = x –

* We have to prove that (2)

Since is a positive definite symmetric matrix, we can diagonalize this matrix by using an orthogonal matrix [denote as U] , S is the diagonal matrix of eigenvalues

* (1)

=

=

* (3)

1. 🡪 | *(4)*

(2)(3)(4) 🡪

1. Find the formula of marginal distribution in Multivariate Gaussian distribution.

The marginal distribution of any subset vector  is also a multivariate normal distribution where μs drops the irrelevant variables from the mean vector μ and Σs drops the corresponding rows and columns from the covariance matrix Σ.

1. Find the formula of conditional distribution in Multivariate Gaussian distribution.